The Semantics of Anaphora

Eastern Generative Grammar

Summer 2024

Todd Snider

1 Getting Started

- This course will be an introduction to some of the puzzles of how to provide a semantics that accounts for anaphora, and some of the approaches to doing so
- We won't be able to cover everything exhaustively, but it'll be a start

1.1 Background

- Taking for granted an introductory level understanding of some syntax and semantics, not more
- If there are terms or concepts here which are unfamiliar to you — ask!

1.2 Aims

- If this week makes you want to take another course somewhere else, or consider how anaphora interacts with some other phenomenon when writing a term paper for another course, I'll consider this a success
- As much as possible, we'll aim for our takeaways to be as theory-neutral as possible: even if you don't want to use ⟨*some framework*⟩, you can learn from the puzzles that motivated it and try to account for them in your preferred framework
  - I won't try to evangelize for any particular formal system
  - We should try to learn from them all, even ones we find unintuitive or unaesthetic

1.3 Definitions

- **Semantics**: the scientific study of the meaning of natural language
- **(a) semantics**: a formal system for the above pursuit, e.g., a logic that assigns meanings to representations of natural language sentences
  - Just like in Logic class we learn about multiple possible logics (and how they differ), in Semantics we consider and compare multiple possible semantic frameworks
  - We evaluate them on the basis of the breadth and accuracy of their empirical coverage (and potential under/overgeneration), their convergence with other theories (including from neighboring fields), the amount/complexity/groundedness of the assumptions they make, their aesthetics, etc.
- **(an) anaphor**: an expression which gets its meaning (partially or in whole) from the interpretation of another expression, its **antecedent**
  - Chomsky 1981 uses the term *anaphor* to the exclusion of pronouns (like *she* or *they*), referring instead only to reflexives (like *himself*) or reciprocals (like *each other*); we will instead use the term broadly, to include pronouns as well
- **anaphora**: this phenomenon, wherein one expression's meaning is determined by another's

- Historically, grammarians distinguished between:
  - **anaphora** (Ancient Greek *aná-* 'up, back' + *phérō* 'to carry'): cases where the antecedent precedes the anaphor

(1) After Joan went to the library, *she* took a nap.

  - **cataphora** (from Ancient Greek *katá-* 'down' + *phérō* 'to carry'): cases where the antecedent follows the anaphor

(2) a. After *she* went to the library, Joan took a nap.
b. *This* is what we need to focus on: climate change.

- We'll treat these as two versions of the same phenomenon: anaphora (with cataphora as a special case)

2 Kinds of Anaphora

- Many different expressions are anaphoric (i.e., make use of anaphora)
- The most commonly discussed anaphoric expressions are pronouns

2.1 By Type

- We can differentiate different classes of anaphoric uses by the type of the **discourse referent** picked out by the antecedent (which is thus also the type of the matching anaphor)

2.1.1 Individual Anaphora

(3) a. Agatha Christie went to the library. *She* started looking at biographies. NAME, SINGULAR IND.
b. Kim and Jessie went to the library. *They* started looking at biographies. NAME, PLURAL IND.
c. A woman went to the library. *She* started looking at biographies. INDEFINITE DESCRIPTION
d. Some women went to the library. *They* started looking at biographies. QUANTIFIER, PL.
e. Every girl in class went to the library, and went straight for *her* favorite book. QUANTIFIER, SG.
f. Nancy Drew went to the library. *She* started looking at biographies. FICTIONAL
g. Joan went to the library. *That* was her favorite place in town. PLACE
h. Joan's book was on the floor. She picked *it* up and put *it* back on the shelf. OBJECT
i. The horse is indigenous to eastern Chile. *It* is a magnificent creature. KIND, cf. Carlson 1977
j. At the academy, Fran learned love for her country. *It* shaped the course of her career. CONCEPT

2.1.2 Event Anaphora

(4) a. Joan went to the library. She had been meaning to do *that* all week.
b. Joan went to the library, and *so* did Marcus.
c. Joan went to the library, and Marcus did *so*, too.
d. Joan went to the library, and Marcus did ε, too.

2.1.3 Propositional Anaphora

(5) a. Joan went to the library. She told me *that*.
b. Joan went to the library, even if Marcus doubts *it*.
c. A: Joan went to the library.
B: *That*'s not true!

2.2 By Anaphor-Antecedent Relationship

- Within types, we categorize anaphora on the basis of how the anaphor and its antecedent are related

2.2.1 Coreference

- The simplest relation is that both the anaphor and antecedent refer to the same entity: they **corefer**
- All of the above examples are of this type
- Even the bound pronoun *her* in (3e), which doesn't properly refer, still has the same relationship with *every girl in the class* (which also doesn't refer)
- Also includes **split reference**, where a plural-marked anaphor picks up entities mentioned separately in the discourse

(6) <u>Joan</u> went to the library, and saw <u>Marcia</u> there. *They* hugged.

  - Here, the anaphor *they* corefers with [the plural individual that we construct, on the fly, out of Joan⊔Marcia], even if we didn't know we needed a representation of said individual until we encountered the plural-marked pronoun (which has to be plural to fit with *hugged*)

2.2.2 Indirect Anaphora

- Indirect anaphora describes cases where an anaphor picks up not the antecedent's discourse referent, but rather an *associate* of that discourse referent.

(7) Myra darted to <u>a phone</u> and picked up *the receiver*. (Webber et al. 2003: (25))

  - *the receiver* picks out the receiver which is *part of* (the entity picked out by) *the phone*

- The term "indirect anaphora" comes from Hellman & Fraurud 1996, but this class has also been called bridging anaphora (Clark 1975; Clark & Marshall 1981; Not et al. 1999), partial anaphora (Luperfoy 1992), textual ellipsis (Hahn et al. 1996), associative anaphora (Cosse 1996), and inferrables (Prince 1992)

2.2.3 Lexically-specified Anaphora

- Lexically-specified anaphora describes cases where an anaphor denotes a function which takes the antecedent (or an associate of the antecedent) as its argument, and returns a referent

(8) I don't like sitting in <u>this room</u>. Can we move *elsewhere*? (Webber et al. 2003: (31))

(9) Sue lifted <u>the receiver</u> as Tom darted to *the other phone*. (Webber et al. 2003: (27))

  - In (8), *elsewhere* denotes a function from [the room picked out by *this room*] to its complement set (within the domain), all of the other places
  - In (9), *other* provides a phone different from [the phone which is the associate of *the receiver*]

- Webber et al. 2003 also lists:
  - "NPs with *such* but no postmodifying *as* phrase"
  - "comparative NPs with no postmodifying *than* phrase"
  - discourse adverbials like *then*, *also*, *otherwise*, *nevertheless*, and *instead*

2.3 Other Anaphoric Processes

- Other processes in natural language have been argued to rely on anaphoric relations, including:
  - the interpretation of tense (Partee 1973, 1984)
  - the interpretation of modality (Stone 1997)
  - response particles (Krifka 2013; Roelofsen & Farkas 2015)
  - discourse relations (Webber et al. 2003)
  - verb phrase ellipsis (Ginzburg & Sag 2000)
  - presupposition (Geurts & Van der Sandt 2004)
- So a coherent theory of anaphora lays the groundwork for many parts of semantic theory, beyond just pronoun interpretation

3 One Starting Point

- Observation 1: Pronouns can have their antecedent either in the same sentence, or in a prior sentence†

(10) Joan went to the library and *she* took out a book.

(11) Joan went to the library. *She* took out a book.

† There may be some restrictions on how far removed the antecedent and anaphor may be from one another, such that the anaphor is still felicitous. And by "may be", I mean: there are. We will come back to those later. But at least in some conditions, **cross-sentential anaphora** is possible

- Observation 2: Pronouns can have non-linguistic antecedents

(12) [We're sitting in a classroom, when suddenly the door is pushed open, and in walks a goat.]
What is *that* doing here?!?

  - In this context, it's entirely unambiguous what *that* refers to: the goat
  - Even though no goat has been mentioned, so there is no linguistic antecedent

- Observation 3: Pronouns can behave as if bound by a quantificational phrase

(3e) Every girl in class went to the library, and went straight for *her* favorite book.

  - The anaphor doesn't pick out any individual, but instead covaries with the quantifier
  - For each girl x picked out by $\forall x$ (part of the translation of *every*), *her* refers to x

- Putting these observations together, many treat **pronouns as variables** (e.g., Heim & Kratzer 1998)

- This is (the semantic) part of a rich tradition going back to (and preceding) the classic Binding Theory
  - For a more detailed history of the development of these ideas, take an advanced Syntax Class
  - Or read Ross 1967; Chomsky 1981, 1982, 1986; Reinhart 1983, and many others
- The motivating idea behind this whole enterprise was to explain the three-way distinction between instances where a potential antecedent and a pronoun *must* corefer, *must not* corefer, and *can optionally* corefer

(13) Zelda bores herself. OBLIGATORY COREFERENCE

(14) Zelda bores her. OBLIGATORY NON-COREFERENCE

(15) Zelda adores her teachers. OPTIONAL COREFERENCE

(Examples from Reinhart 1983)

- And explaining all of the impossible combinations of the above

- This is what led to the Binding Principles (Chomsky 1981):

  Principle A: An anaphor [i.e., a reflexive or reciprocal] must be bound in its governing category [i.e., in its clause]

  Principle B: A pronoun must be free [i.e., unbound] within its governing category

  Principle C: An R-expression [i.e., a name or a definite description] must be free

- These continue to be discussed to this day, even though the theory surrounding them has changed

- But let's get back to where this leaves us in a modern setup like that of Heim & Kratzer 1998
  - Pronouns are at their core just variables; all the variation among them is just phonology
    - in other words, Agreement with [+sg, +fem] φ-features gets spelled out as *she*
    - But as far as their semantics are concerned, they at most carry definedness conditions if those φ-features trigger presuppositions
    - Beyond that...
  - $[\![\text{he}]\!]^{\mathcal{M},g} = [\![\text{she}]\!]^{\mathcal{M},g} = [\![\text{it}]\!]^{\mathcal{M},g} = [\![\text{they}]\!]^{\mathcal{M},g} = \ldots$
  - Traces and Pronouns Rule
    If α is a pronoun or a trace, g is a $\mathcal{M}$-assignment, and $i \in dom(g)$, then $[\![\alpha_i]\!]^{\mathcal{M},g} = g(i)$.
  - When we interpret sentences, pronouns (and traces) already come packaged with an (unpronounced) **index**
  - This index tells us, in effect, how to interpret the pronoun, and which other expressions it is **coindexed** with
    - If multiple pronouns refer to the same entity, they get the same index
    - A variable binder and its bound variables get the same index (sometimes written as $binder^1$ and $variable_1$)
  - The variable assignment g is a partial function which takes indices (in the domain of $\mathbb{N}$) and returns entities in the domain (of individuals) of the model
  - When a pronoun is bound, the binder modifies the assignment such that the pronoun('s index) is mapped to whoever it 'should'
  - When a pronoun is unbound, the (unchanged) assignment maps the pronoun('s index) to whatever it otherwise would have
    - This will be the case for cross-sentential anaphora, and for non-linguistic antecedents, as in neither case do we have proper binding
  - In either case, if the assignment is doing its job, and we have a well-defined context, then the pronoun will be mapped to the right entity in the domain of discourse
  - Variables are bound under only certain structural conditions:

(16) β binds α if and only if:

  i. α is an occurrence of a variable,

  ii. β is an occurrence of a variable binder,

  iii. β c-commands α,

  iv. β is co-indexed with α,

  v. β does not c-command any other variable binder occurrence which also c-commands and is co-indexed with α.

- ? How does the assignment gets its starting definition?
  - ∗ Answer 1: It starts blank, and the only mappings well-defined are those which involve binding or lambda abstraction to modify the (empty) assignment
  - ∗ Answer 2: ¯_(ツ)_/¯ That's pragmatics, not the job of semantics
- ? Isn't interpreting only sentences with invisible indexes cheating?
  - ∗ Maybe! A reason to prefer a theory without it, then...
  - ∗ But maybe not, if we think syntactic structure has a role to play, and that it constrains the possible interpretations; nobody thinks the indexes are really 'there', but they are representational stand-ins for the information provided to the interpretive module (semantics) by the structural module (syntax)

- This leads us to two very famous puzzles surrounding binding and anaphora

4 Picture NPs

4.1 The Puzzle

- **Picture NPs** are a class of observed exception to (i.e., problems for) the classic picture presented by Binding Theory
- Principle A says that (reflexive and reciprocal) anaphors have to be locally bound,
  and Principle B says that pronouns must be locally free
- A natural consequence of this is that they shouldn't show up in the same places: they should be in complementary distribution
- Picture NPs show that this isn't always the case

(17) Joan$_i$ finally saw a picture of her$_i$/herself$_i$.

(18) Joan$_i$ thinks that pictures of her$_i$/herself$_i$ will sell.

(19) Joan$_i$ saw a snake near her$_i$/herself$_i$.

- Picture NPs often make use of a representational NP, a content-bearing NP like *picture*, *book*, *painting*
  - Crucially, these are NPs that can take (nominal) arguments with a preposition like *of* or *about*
  - They can also show up with a possessor argument: *Alex's book*, *Bobby's painting*
- This class of counterexamples also includes examples like (19) that don't have any specific kind of NP, but they do still include a preposition before the anaphor
- In either case, there seems to be something about this structure which licenses both a pronoun (*her*) and a reflexive anaphor (*herself*)

4.2 Approaches to Resolving the Puzzle

- For Chomsky 1986, the answer lies in treating the governing category (≡ the binding domain) of pronouns as distinct from the governing category of (reflexive and reciprocal) anaphors
  - Each have to be bound/free relative to their governing category
  - But if they have different rules about what constitutes a governing category for them, then they can have different rules in the same context
- For a reflexive or reciprocal anaphor, the governing domain is the whole sentence
  - So in (17), the governing category is the whole sentence

- So *herself* is properly bound by Joan
- So Principle A is observed

- For a pronoun, the governing category is the NP
  - So in (17) the governing category is [*a picture of her*]
  - So *her* is unbound within this domain
  - So Principle B is observed
- Similar ideas about asymmetries of binding domains have been incorporated into LFG treatments of these kinds of examples (Dalrymple 1993, 2001; Bresnan 2001)
- Meanwhile in HPSG, the equivalent of Principle A is weakened by a notion of *exemption*: that reflexive and reciprocal anaphors only have to be bound in certain conditions (i.e., when there is a local *coargument*) (Pollard & Sag 1992, 1994; Asudeh 1998)
- But these approaches struggle with further data

(20) With a syntactic possessor:
  a. Joan$_i$ finally saw Hank's picture of her$_i$.
  b. Joan$_i$ finally saw Hank's picture of herself$_i$.

(21) Without a syntactic possessor:
  a. * Joan$_i$ finally painted a picture of her$_i$.
  b. Joan$_i$ finally painted a picture of herself$_i$.

- Theories that pull apart the governing categories (≡ binding domains) allow both pronouns and (reflexive and reciprocal anaphors) in the same places, across the board
  - So they make the right predictions for (20a),(20b),(20b)
  - But they wrongly predict (21a) to be grammatical
- Theories that use exemption require (reflexive and reciprocal) anaphors to be bound when there is a coargument
  - So they wrongly predict (20b) to be ungrammatical
  - And they don't explain why (21a) is ungrammatical (predicting it to be grammatical)
- Later works account for these complications by requiring more fine-grained analyses of argument structure (e.g., Jaeger 2004 argues that the agentivity of the binder plays a role)
- What is the 'final', correct solution to this puzzle? TBD! (Get used to this)

5 Donkey Sentences

- **Donkey sentences** (sentences involving **donkey anaphora**) also pose a challenge to the classical Binding Theory
- But they pose a few other challenges as well
- Originally discussed in Geach 1962; Kamp 1981, among many others
- Donkey sentences are so-called because the most classic example(s) involve, well, you can guess

(22) If Pedro owns a donkey, he beats *it*. (Partee 1984: (6a))

(23) Every farmer who owns a donkey beats *it*. (Partee 1984: (6b))

- For the sake of history and continuity, it's worth knowing what the original examples are
- But it's 2024, we don't need our example sentences to revolve around violence (or animal cruelty) when none is required

(24) Every farmer who owns a donkey loves *it*.

- It's very clear what the interpretation of this sentence is: each owned donkey is loved by their owner

5.1 Puzzle One: Binding Domain

- Donkey sentences pose a challenge to the classic Binding Theory, because while it's clear that the only possible (and sensible) antecedent for the pronoun *it* is the DP *a donkey*, they're not in the right structural relationship for *a donkey* to bind the pronoun
- Remember that in order for β to bind α, β must c-command α (among other constraints)
- But *a donkey* is in a relative clause, it isn't a sister to the VP (containing the pronoun)
- So *a donkey* doesn't c-command *it*
- So *a donkey* can't bind *it*
- Semantically, *it* behaves as though bound by *a donkey*, but it cannot be syntactically bound

5.2 Puzzle Two: Translating the Indefinite

- Normally, we translate indefinites using existential quantification

(25) A man walked in. $\rightsquigarrow$ $\exists x\,(\texttt{Man}(x) \wedge \texttt{WalkIn}(x))$

(26) Alex saw a dog. $\rightsquigarrow$ $\exists x\,(\texttt{Dog}(x) \wedge \texttt{Saw}(\texttt{alex}, x))$

- But existential quantification is usually thought too weak to properly capture the meaning of (24)

(24) Every farmer who owns a donkey loves *it*.

  - If Charlie the farmer owns two donkeys, under which scenarios is (24) true?
  - Does Charlie have to love both of them? Or is loving just one sufficient?

- Most people think the former: in other words, we want to assign a paraphrase like 'every farmer loves every donkey that they own'
- In other words, we want to translate *a donkey* not with ∃ but with ∀
- That's not impossible, but it is confusing! Are all indefinites systematically ambiguous, between an existential and a universal reading?
- We might be tempted to try to explain this universal flavor as somehow being a consequence of the indefinite being within the scope of another universal quantifier: the one denoted by *every farmer*
  - Maybe we have something like 'scopal harmony'?
  - The indefinite gets an existential interpretation when it is on its own, but gets strengthened to a universal interpretation when part of the restrictor of a universal quantifier?
- But that attempted explanation won't work, because there are other sentences with the exact same syntactic and scopal structure, but that don't get a universal interpretation

(27) Every student who owns an umbrella brings *it* to campus when it rains.

  - It's clear that the antecedent for *it* has to be *an umbrella*, despite the lack of c-command
  - If a student owns two umbrellas, under which scenarios is (27) true?
  - Do they have to bring both to campus, for the sentence to be true?

- Most people think this sentence is true just so long as a single umbrella is brought to campus: a student doesn't have to bring every umbrella they own to campus
- Even though the farmers have to feed every donkey!
- So whether an indefinite is translated as an existential quantifier or a universal quantifier is informed at least partially by context, it seems?

5.3 Puzzle Three: The Scope of the Indefinite

- However we pick out the right quantifier, we know that we get what amounts to a conditional interpretation of (24)
- (24) $\equiv$ 'If a farmer owns a donkey, they love it'
- If we wanted to translate this into Predicate Logic, we'd end up with:
  $\forall x\,((\texttt{Farmer}(x) \wedge \exists y\,(\texttt{Donkey}(y) \wedge \texttt{Own}(x,y))) \rightarrow \texttt{Love}(x,y))$
- But this isn't a well-formed sentence of Predicate Logic, because the y in $\texttt{Love}(x,y)$ is not bound
- In a sense, this is an echo of Puzzle One, as it plays out in the formal translation of (24)
- To get the interpretation we want, we have to 'hold open' or extend the scope of (the quantifier denoted by) the indefinite, to include the translation of the consequent
  $\forall x\,(\texttt{Farmer}(x) \wedge \exists y\,(\texttt{Donkey}(y) \wedge \texttt{Own}(x,y) \rightarrow \texttt{Love}(x,y)))$
- But why do we get to do that? What licenses this move? We would need to explain how this works
- And this translation ends up wrong for another reason:
  - What is the antecedent of the conditional?
  - As (re)written, we have two choices:
    * $\texttt{Own}(x,y)$ or
    * $\texttt{Donkey}(y) \wedge \texttt{Own}(x,y)$
    * It can't include the y binder, or else either (i) the consequent will be outside of its scope or (ii) the parentheses won't be properly nested
  - That means that we actually have a much stronger meaning for this translation than we want: 'Every x is a farmer, and, *if* there is some donkey y that x owns, then x loves y.
  - This is way too strong! It requires every entity x in the domain to be a farmer!
    (Including the donkey!)
- Normally, we have an extra set of parentheses around the restrictor of the universal quantifier (the antecedent of the conditional), so that we get
  - 'For every x, *if* x is a farmer who owns a donkey y, then x loves y'

  instead of
  - 'For every x, x is a farmer and if...'
- In our first translation attempt:
  $\forall x\,((\texttt{Farmer}(x) \wedge \exists y\,(\texttt{Donkey}(y) \wedge \texttt{Own}(x,y))\,) \rightarrow \texttt{Love}(x,y))$
- But when we extend the scope of $\exists y$ to include the consequent, we have to remove this crucial set of parentheses
- So even if we know which quantifier to choose, and even if we have the ability to arbitrarily extend the scope of that quantifier, it'll get us into trouble in other ways

5.4 One Approach to Resolving these Puzzles: E- and D-type Approaches

- E-type approaches, and later, D-type approaches, were motivated by thinking seriously about the paraphrases we might assign to donkey sentences when being careful about them (Evans 1977)
- If you wanted to paraphrase away the *it*, i.e., translate the sentence (24) into a metalanguage without using pronouns, how would you do it?

(24) Every farmer who owns a donkey loves *it*.

(24′) Every farmer who owns a donkey loves the donkey(s) that they own.

(24″) Every farmer who owns a donkey loves the donkey(s) that that farmer owns.

- The idea behind E- and D-type approaches, roughly: give pronouns like *it* the semantics, and even the syntax(!), of the definite descriptions that they are equivalent to
- This hearkens back to Hankamer & Sag's (1976) distinction between deep & surface anaphora
  - **deep anaphors** are those which are, at their core, anaphors, and so are "pragmatically controlled" (i.e., valued by context, and so might have non-linguistic antecedents, etc.)
  - **surface anaphors** are those which are anaphors only in virtue of being "syntactically controlled" by their binders (and so, like ellipsis sites, are related to their antecedents by an identity condition and get reduced phonologically to a mere pronoun, or to ε)
- If these (surface) pronouns only show up (on the surface) as pronouns in virtue of being a phonologically reduced version of (at their core) a more substantive description,
- then perhaps we should interpret them not just as bare variables (whose scope is in question, etc.),
- but as full-fledged definite descriptions
- This way, we might still have to worry about the right DPs being coindexed, but we don't have the same worries about scope or binding
- This idea might also be thought of as akin to the Copy Theory of Movement, still quite alive in many modern Chomskyan syntactic theories
  - When you move a constituent, you don't actually move it from its original position
  - Instead, you copy it, leaving behind the original copy in the original position (which we interpret semantically as a trace)
  - Often, we just pronounce the copy which is structurally highest
  - In some circumstances, in some languages, other copies (or even, all copies) get pronounced
  - This is what happens with some resumptive pronouns, for example

(28) a. This is the woman who I know you'll like *t* when you meet *t*.
b. This is the woman who I know you'll like her when you meet her.

(29) Yesh li rak sheela axat she+ ani lo macliax le+havin (ota).
COP 1.SG.DAT only question one that 1.SG.NOM NEG succeed to+understand it
'There is only one question which I cannot understand.' (Hebrew, Ariel 1999)

- These approaches aren't currently very much in vogue
- Hence my trying to give their motivation a little boost
- One reason that some researchers looked for a different approach, is that there are some sentences which make replacement-with-a-description challenging

6 Bishop Sentences

- This objection against E- and D-type approaches is called *the problem of indistinguishable participants*, and originates from Heim 1990 (citing data from Hans Kamp)
- But we semanticists love naming phenomena after their famous first example sentences, so: **bishop sentences**

(30) If a bishop meets a(nother) bishop, he always blesses him.

(31) Whenever a bishop meets a(nother) bishop, he blesses him.

- Like donkey sentences, here we have indefinite descriptions behaving denoting universal quantifiers
  - In (30), this is facilitated by *always*; in (31), by *whenever*
- Because of course, bishop sentences *are* donkey sentences: they have semantic binding behavior but no syntactic binding
- But they crucially are donkey sentences with indistinguishable participants: ones where we can't tell the participants apart from one another
- Consider the same exercise we did with (24), trying to replace each pronoun with a suitable definite description: how would you replace *he* and *him* in (30)?
  - We can't use *the bishop*, because we have two bishops! No contextual uniqueness
  - We can't use *the bishop who meets a bishop*, because that similarly describes both participants
- This isn't necessarily a knock-down argument: there are ways to augment theories that treat pronouns as definite descriptions so as to differentiate among 'indistinguishable' participants
- But it's not simple or obvious!
- There are responses to this objection and further development of D-type theories (Elbourne 2005; Kroll 2008; Elbourne 2010, among others)

Checking In

- There is a family of other approaches which we could pivot to now
- But before we do that, it'll be useful to ask another question, and see a few more puzzles
- Then we'll take a whirlwind tour of a bunch of formalisms

Questions We Already Know We Might Want To Ask
⇝ Desiderata For a Semantics of Anaphora

- Can it handle anaphora of different types?
  - (i.e., individuals, events, propositions)
  - (i.e., coreference, indirect, lexically-specified)
- How does it handle cross-sentential anaphora?
- How does it handle non-linguistic antecedents?
- How does it deal with the scope of donkey anaphora?
- How does it deal with indistinguishable participants (i.e., bishops)?

7 Introducing Discourse Referents

- One question we might ask—not about semantic formalisms, but about anaphora broadly—is:
How are discourse referents introduced (into our representation of the discourse) (such that we can then anaphorically refer back to them, if we want to)?
  - Even while we recognize that some special non-linguistic occurrences (maybe, highly salient ones? like a goat walking in when we don't expect goats in classrooms) might make them available as well, we know discourse reference are also (and possibly most of the time) introduced by linguistic material
  - So we can focus on finding out the circumstances under which that happens with linguistic material, even if we know we aren't providing an exhaustive answer to the question
- One reasonable intuition we might have:
  - We get discourse referents when things have been mentioned
  - If we mention something, we can refer back to it
- Most of the time, this prediction is borne out: name something, and you can refer back to it
- But there are some examples which don't comport with it

(32) a. Bill has a car. (Karttunen 1969: (3a))
b. *It* is black. (Karttunen 1969: (3b))

(33) a. Bill doesn't have a car. (Karttunen 1969: (4a))
b. # *It* is black. (Karttunen 1969: (4b))

  - In both discourses, we have the indefinite DP *a car*
  - In (32), we get a licit use of the pronoun *it* to refer back to it
  - But in (33), that same attempted reference fails
- What explains this discrepancy?

- You probably have a strong intuition: In (33), there is no car! (So of course we can't refer back to it!)
- This is part of the data which motivates Karttunen's (1969) analysis
  - This is the paper which introduced the term *discourse referent*
  - Its main aim was answering the question introduced at the top of this section:
When are discourse referents introduced?
- Karttunen's (1969) answer: "[a] non-specific indefinite NP in an affirmative sentence (single sentence or a complement) establishes a[n individual] discourse referent just in case the proposition represented by the sentence is asserted, implied or presupposed by the speaker to be true"
  - So in (32), the proposition 'Bill has a car' is "**asserted**, implied or presupposed by the speaker to be true"
  - While in (33), the proposition 'Bill has a car' is not only not asserted, implied or presupposed by the speaker to be true", but in fact is explicitly asserted by the speaker **not** to be true
  - So this generalization captures the data in (32)–(33) (as well as the other data in the paper)
- But here are three puzzles which raise complications for this nice, clean picture

7.1 Sub-parts of Words

- This class of examples doesn't have a specific catchy name, but they all involve attempted anaphora to [antecedents which are sub-parts of words]

(34) a. Fritz owns a dog and *it* bites him.
b. # Fritz is a dog-owner and *it* bites him. (cf. Evans 1977)

(35) a. Followers of McCarthy are now puzzled by *his* intentions.
b. # McCarthyites are now puzzled by *his* intentions. (Postal 1969)

  - In (34a), we get licit anaphora to the discourse referent associated with *a dog*
  - The first clause of (34b) also includes the string *a dog*!
  - AND it passes Karttunen's (1969) test (i.e., it doesn't fail for the same reason that Bill's non-existent car in (33) fails), because the speaker of (34b) *does* "assert[], impl[y], or presuppose[]" that the proposition is true
    * The speaker of (34) asserts that 'Fritz is a dog-owner' is true
    * The speaker of (34) presupposes that 'Fritz owns a dog' is true
    * Whichever 'host' proposition we're interested in, this is consistent with Karttunen's (1969) generalization
  - But nevertheless, the attempted anaphoric reference of *it* to the dog Fritz owns (entailed by Fritz being a dog-owner) fails

  - You might think this is because *dog-owner* is compatible with Fritz owning multiple dogs
    * So we might accept the entailed proposition 'Fritz owns at least one dog'
    * But the context might not yet support the presupposition of 'Fritz owns only one dog', which we might argue is conveyed by the use of (the singular) *it*
  - But this is true of *a dog* in (34a), too: it is compatible with Fritz owning multiple dogs ("In fact, he owns three!")
  - And also, why shouldn't be able to just accommodate that presupposition in (35b), the way we seem to in (34)?!

  - The same sort of thing happens in (35):
    * Both versions include the string *McCarthy*
    * In both sentences, the speaker is committed to the truth of either relevant proposition
    * Nevertheless, the *McCarthy* within *McCarhyites* seems not to license anaphora

7.2 Partee's Marbles

- As a reminder, Karttunen's (1969) generalization is about whether 'host' sentence (containing the indefinite NP) is "asserted, implies, or presupposed by the speaker to be true"

  - To make sure we don't get introduction of discourse referents under negation, or out from inside conditionals, etc.

- Partee's marbles example doesn't directly contradict Karttunen's (1969) generalization, because the 'failure case' doesn't involve an indefinite NP antecedent at all

- Nevertheless, it's useful to include here, for two reasons

(36) a. One of the ten balls is missing from the bag.
It's under the couch.

b. # Nine of the ten balls are in the bag.
   It's under the couch. (Partee 1989)

- (The original example from Partee 1989 uses balls, but it is usually reproduced with marbles, maybe because it's easier to imagine them rolling under a couch than some unspecified type of ball; the original "balls" has been memory-holed, so I'm referring to them here as they tend to be referred to these days)
- This example was originally presented as an argument against a potential answer, that existence+**salience** is a sufficient criterion for discourse referent introduction
- We might have been tempted to go in that direction, because of Karttunen 1969+our goat example (with a highly salient goat!)
- But, Partee's marbles show us, that is not sufficient: even when we know the speaker is committed to the truth of the existence of the 10th marble, and even when its location is the topic of conversation, and so this marble is highly salient, we *still* can't refer to it anaphorically, without some explicitly mentioned DP to refer back to

- These two puzzles, taken together, are (among the) motivations for what has been called the **Formal Link Condition** (Postal 1969; Kadmon 1987; Heim 1990, a.o.)
  - The FLC requires that a pronoun (i) have an overt NP antecedent, and (ii) that this antecedent cannot be a sub-part of a word
  - This condition explains the infelicity of anaphoric reference to individuals which are salient (but unmentioned) in a discourse (as in (36b)), as well as to individuals whose existence is entailed but who are not named explicitly (as in (34b) and (35b))

7.3 Partee's Bathroom

- One more example, also from Barbara Partee, also shows us that there's more going on than is captured by Karttunen's (1969) generalization

(37) [*touring a house*:]
Either there is no <u>bathroom</u>, or *it* is in a weird place.

- Here we *do* get licit anaphoric reference, even though it isn't predicted by Karttunen 1969
- The antecedent(?) associated with *bathroom* is *not* an indefinite NP, and it's *not* inside a clause whose truth the speaker is asserting/implying/presupposing to be true!
  - For disjunction like this, the speaker is (usually thought to be) committed to the *possibility* of the truth of each disjunct, but not to the truth of any disjunct individually
    - ∗ Plus some ignorance inferences, etc., which don't matter to us here
  - And *bathroom* is under negation! There is no bathroom!! (...at least possibly not!)
- There are other such cases like this
- And potential explanations, which get us into thinking about
  - the 'alternatives' associated with disjunction (visible especially with imperatives, cf. Veltman 1996)
  - modal subordination (e.g., Roberts 1989)

  but we don't have the time to get into those in depth right now
- For now, its just worth keeping this in mind, as we further problematize where discourse referents even come from

8 A Whirlwind Tour of Semantic Frameworks for Anaphora

8.1 Dynamic Predicate Logic (DPL; Groenendijk & Stokhof 1991)

- In our standard static first-order predicate logic (PL):
  - Anaphors are variables
  - Sentences are interpreted with respect to a model (which contains the entities and predicates relevant for interpretation) and an assignment function (which assigns variables to entities)
- For example, a sentence like (38a) is interpreted as in (38b), relative to a model $\mathcal{M}$ and an assignment function g

(38) a. She napped.
b. $[\![\text{napped}(x)]\!]^{\mathcal{M},g}$

- This sentence will be true if and only if the individual that g maps to x did in fact nap in the model
- Given a model, we can consider a sentence to denote the set of assignment functions that verify it

- Dynamic Predicate Logic (DPL; Groenendijk & Stokhof 1991) was created to account for cross-sentential anaphora and donkey-sentences, and—as anaphors are considered to be (syntactically free) variables—the binding of variables
- In DPL, a sentence denotes not a set of assignment functions but a *relation* between sets of assignment functions, represented as a set of ordered pairs of assignment functions: input/output pairs of assignment functions
- Formulas with the existential quantifier $\exists x$ change the variable assignment for x in the output: they change the assignment function in terms of what it assigns x to
- Formulas without the existential quantifier, including atomic formulas, are tests on input/output pairs: they don't change variable assignment in any way, but they ensure that the assignment pair verifies the formula (just as in static PL)
- One other key change: In DPL, variable binding of $\exists x$ can continue indefinitely rightward, including across sentence boundaries, unless it is closed by a closure operator
  - So the sentences in (39a) and (40a) are translated into DPL as in (39b) and (40b), respectively
  - And the discourse in (41a) is translated into DPL as in (41b)

(39) a. A man walks in the park.
b. $\exists x[\text{man}(x) \wedge \text{walk\_in\_the\_park}(x)]$

(40) a. He whistles.
b. $\text{whistle}(x)$

(41) a. A man walks in the park. He whistles.
b. $\exists x[\text{man}(x) \wedge \text{walk\_in\_the\_park}(x)] \wedge \text{whistle}(x)$ (Groenendijk & Stokhof 1991)

  - Even though the final x in (41b) is not contained within the brackets, it is still dynamically bound by the existential quantifier, so we interpret *he* as referring to the man walking in the park

- DPL is compositional at the sentence level: the same DPL translation is assigned to *He whistles*, regardless of any prior discourse context
- But it isn't compositional at a sub-sentential level (like our Montague semantics)
- It is the dynamic binding of DPL that allows the pronoun to be interpreted sensitive to context

- In DPL, the meaning of a sentence is not just its truth conditions, but also its dynamic potential: its (in)ability to bind variables in subsequent discourse
- Truth conditions are "an essential ingredient of meaning", but "truth conditions do not exhaust meaning" (Groenendijk & Stokhof 1991: 98)
- Even though the first sentences in (36a) and (36b) are truth-conditionally equivalent, they have different anaphoric effects: only (36a) allows for felicitous follow-up using the anaphor *it*
- On a DPL account, only (36a) can 'capture' the variable denoted by *it* because (36a)—but not (36b)—contains an existential quantifier
- The two sentences are truth conditionally equivalent, but their dynamic potential is different

Figure 1: Fragment of Dynamic Predicate Logic (Groenendijk & Stokhof 1991)

VOCABULARY

- n-place predicates
- individual constants
- individual variables
- negation $\neg$
- conjunction $\wedge$
- disjunction $\vee$
- implication $\rightarrow$
- existential $\exists$
- universal $\forall$
- identity $=$

SYNTAX

1. If $t_1, \ldots, t_n$ are individual constants or variables, R is an n-place predicate, then $Rt_1 \ldots t_n$ is a formula.
2. If t_1 and t_2 are individual constants or variables, then $t_1 = t_2$ is a formula.
3. If ϕ is a formula, then $\neg\phi$ is a formula.
4. If ϕ and ψ are formulas, then $[\phi \wedge \psi]$ is a formula.
5. If ϕ and ψ are formulas, then $[\phi \vee \psi]$ is a formula.
6. If ϕ and ψ are formulas, then $[\phi \rightarrow \psi]$ is a formula.
7. If ϕ is a formula, and x is a variable, then $\exists x\phi$ is a formula.
8. If ϕ is a formula, and x is a variable, then $\forall x\phi$ is a formula.
9. Nothing is a formula except on the basis of 1–8.

SEMANTICS

A model M is a pair $\langle D, F\rangle$, where D is a non-empty set of individuals, F an interpretation function, having as its domain the individual constants and predicates. If α is an individual constant, then $F(\alpha) \in D$; if α is an n-place predicate, then $F(\alpha) \subseteq D^n$. An assignment g is a function assigning an individual to each variable: $g(x) \in D$. G is the set of all assignment functions. Next, we define $[\![t]\!]_g = g(t)$ if t is a variable, and $[\![t]\!]_g = F(t)$ if t is an individual constant.

1. $[\![Rt_1 \ldots t_n]\!] = \{\langle g, h\rangle | h = g \;\&\; \langle [\![t_1]\!]_h \ldots [\![t_n]\!]_h\rangle \in F(R)\}$.
2. $[\![t_1 = t_2]\!] = \{\langle g, h\rangle | h = g \;\&\; [\![t_1]\!]_h = [\![t_2]\!]_h\}$.
3. $[\![\neg\phi]\!] = \{\langle g, h\rangle | h = g \;\&\; \neg\exists k : \langle h, k\rangle \in [\![\phi]\!]\}$.
4. $[\![\phi \wedge \psi]\!] = \{\langle g, h\rangle | \exists k : \langle g, k\rangle \in [\![\phi]\!] \;\&\; \langle k, h\rangle \in [\![\psi]\!]\}$.
5. $[\![\phi \vee \psi]\!] = \{\langle g, h\rangle | h = g \;\&\; \exists k : \langle h, k\rangle \in [\![\phi]\!] \vee \langle h, k\rangle \in [\![\psi]\!]\}$.
6. $[\![\phi \rightarrow \psi]\!] = \{\langle g, h\rangle | h = g \;\&\; \forall k : \langle h, k\rangle \in [\![\phi]\!] \Longrightarrow \exists j : \langle k, j\rangle \in [\![\psi]\!]\}$.
7. $[\![\exists x\phi]\!] = \{\langle g, h\rangle | \exists k : k[x]g \;\&\; \langle k, h\rangle \in [\![\phi]\!]\}$.
8. $[\![\forall x\phi]\!] = \{\langle g, h\rangle | h = g \;\&\; \forall k : k[x]h \Longrightarrow \exists j : \langle k, j\rangle \in [\![\phi]\!]\}$.

ϕ is *true with respect to g in M* iff $\exists h : \langle g, h\rangle \in [\![\phi]\!]_M$.
ϕ is *valid* iff $\forall M \forall g : \phi$ is true with respect to g in M.
ϕ is a *contradiction* if $\forall M \forall g : \phi$ is false with respect to g in M.

8.2 Discourse Representation Theory (Kamp 1981; Asher 1993)

- In Discourse Representation Theory (DRT; Kamp 1981) and an extension thereof, Segmented Discourse Representation Theory (SDRT; Asher 1993), sentences are translated into discourse representation structures (DRSs)
- Each DRS consists of:
  - A universe of discourse referents
  - A set of conditions
- DRSs are frequently presented as boxes, with the list of discourse referents on the top line of the box, and the conditions listed below

(42) a. A boy kicked Fred. (Asher 1993: 66, (1))

b. (Asher 1993: 66, (K1))

x, y
boy(x) Fred(y) kick(x,y)

- Sequences of sentences are combined via DRS-merge, which unions both the universes and conditions of the DRSs it combines
- Anaphors are translated as variables awaiting assignment, which happens after DRS construction, via anaphora resolution

(43) a. A boy kicked Fred. He cried. (Asher 1993: 67, (3))

b. (Asher 1993: 67, (K3))

x, y, z
boy(x) Fred(y) kick(x,y) cry(z) z = ?

- The anaphor *he* in (43a) is translated as the variable z in (43b), not yet linked to any antecedent
- After anaphora resolution, the DRS for (43a) would be as in (43c)

(43) c. (Asher 1993: 68, (K′3))

x, y, z
boy(x) Fred(y) kick(x,y) cry(z) z = y

- DRSs can be embedded inside one another (making one a subDRS of the other)
- DRT translations of quantifiers, conditionals, and certain embedded clauses make use of this feature
- For example, the donkey sentence in (44a) is translated into the DRS in (44b) prior to anaphora resolution

(44) a. If a high school student owns a mustang, he works on it every day. (Asher 1993: 76, (6))

b. (Asher 1993: 76, (K6))

x, y	⇒	z_1, z_2
high-school student(x) Mustang(y) own(x, y)	⇒	z_1 = ? z_2 = ? work-on-every-day(z_1, z_2)

- The structural configuration of how subDRSs are embedded has consequences for anaphora resolution in DRT: Only certain universes of discourse referents are accessible from one (sub)DRS to another
  - For instance, discourse referents in the universe of the antecedent of a conditional are available to its consequent
  - This is what allows z_1 and z_2 from (44b) to be identified with x and y
  - But the discourse referents introduced in a consequent are not accessible from the conditional antecedent

Figure 2: Fragment of Discourse Representation Theory (Asher 1993)

VOCABULARY

Object language symbols:

$x, y, x_1, \ldots$	individual, eventuality, or plural discourse referent variables
$e, e_1, \ldots$	eventuality discourse referent variables
P, Q, P_1, Q_1, ...	predicative DRS variables
u, x, y, z, x_1, ...	discourse referents (individual type)
loves(,), boy( ), etc.	DRT predicates
$\Rightarrow, \neg, \vee$	DRS operators

Metalinguistic symbols:

$\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$, $\mathbf{x}_1$, ...	variables over individual, eventuality, or plural discourse referents Given the above:
$\mathbf{e}$, $\mathbf{e}_1$, ...	variables over eventuality discourse referents
$\mathbf{t}_1$, $\mathbf{e}_2$, ...	variables over temporal discourse referents
$\mathbf{x}, \mathbf{y}, \ldots$	variables over discourse referent variables
$\mathbf{e}, \mathbf{e_1}, \ldots$	variables over eventuality discourse referent variables
$\mathbf{K}$, $\mathbf{K}'$, $\mathbf{K}_1$, ...	variables over DRSs
$\mathbf{P}$, $\mathbf{Q}$, $\mathbf{Q}_1$, ...	variables over predicative DRS variables
φ, ψ, ζ	variables over DRT predicates
$\mathbf{\Phi}, \mathbf{\Psi}, \mathbf{\Phi_1}, \ldots$	variables over DRT conditions and sets of DRTS conditions

(i) If ψ is n-ary, then $\psi(\mathbf{x}_1, \ldots, \mathbf{x}_n)$ is an atomic condition.

(ii) $\mathbf{x}_1 = \mathbf{x}_2$, $\mathbf{e}_1 < \mathbf{e}_2$, $\mathbf{e}_1 < \mathbf{t}$ are conditions.

(iii) If ψ is a quantifier relation symbol, then the following are conditions: $\neg$K, $\mathrm{K}_1 \Rightarrow \mathrm{K}_2$, $\mathbf{e}$-K, and $\mathrm{K}_1 \left\langle \overset{\psi}{\mathrm{x}} \right\rangle \mathrm{K}_2$.

(iv) If $\mathbf{x}$ is a discourse referent in U_K, then $\mathbf{X} = \Sigma_\mathbf{x}\mathrm{K}$ is a condition.

(v) $\mathbf{X} = \mathbf{x}_1 + \ldots + \mathbf{x}_n$ is a condition.

(vi) A DRS is a pair $\langle \mathrm{U}, \mathrm{Con} \rangle$, where U is a set of discourse referents, and Con a set of conditions.

Conditions that contains subDRSs are complex conditions; those without constituent conditions are simple or atomic conditions. An atomic DRS contains only atomic conditions; otherwise it is complex.

Subordination: If K is a constituent of a complex condition of K$'$, then K is subordinate to K$'$. If K_n is a constituent of a complex condition of K_{n-1} and K_{n-1} is subordinate to K, then K_n is subordinate to K.

Accessibility: Let K = K_0 or K be subordinate to K_0. Then:

(i) If $\mathbf{x}$, $\mathbf{y} \in \mathrm{U}_\mathrm{K}$, then $\mathbf{y}$ is accessible to $\mathbf{x}$ in K_0;

(ii) if K$'$ is subordinate to K and $\mathbf{x} \in \mathrm{U}_{\mathrm{K}'}$ and $\mathbf{y} \in \mathrm{U}_\mathrm{K}$, then $\mathbf{y}$ is accessible to $\mathbf{x}$ in K_0;

(iii) if $\mathbf{y} \in \mathrm{U}_{\mathrm{K}'}$, $\mathbf{x} \in \mathrm{U}_{\mathrm{K}''}$ and $\mathrm{K}' \Rightarrow \mathrm{K}''$ is a complex condition of K, then $\mathbf{y}$ is accessible to $\mathbf{x}$ in K_0;

(iv) otherwise, $\mathbf{y}$ is not accessible to $\mathbf{x}$ in K_0.

Figure 2, continued

SEMANTICS

An intentional DRS model M is a quintuple $\langle$ W, T, D, E, $[\![\ ]\!]\rangle$.

(i) W and T are non-empty sets (of worlds and times respectively).

(ii) D is a function from W $\times$ T into a non-empty, atomic lattice ($\mathrm{D}_{\langle \mathrm{w,t}\rangle}$ is the "domain of individuals and sums of individuals in w at t").

(iii) E is a function from W $\times$ T into a set of objects (eventualities) partially well ordered by inclusion such that:

(a) if ϕ is an E-path description, $\forall \mathrm{e}_1 \in \mathrm{E}_{\langle \mathrm{w,t}_1\rangle} \forall \mathrm{e}_2 \in \mathrm{E}_{\langle \mathrm{w,t}_2\rangle} \ldots \forall \mathrm{e}_n \in \mathrm{E}_{\langle \mathrm{w,t}_n\rangle}(\varphi(\mathrm{e}_1, \ldots \mathrm{e}_n) \rightarrow \exists \mathrm{e}' \in \mathrm{E}_{\langle \mathrm{w,t}_n\rangle}(\mathrm{e}_1 \subset \mathrm{e}\& \ldots \& \mathrm{e}_n \subset \mathrm{e}')$ and

(b) if ψ is an eventuality description,
then $\exists \mathrm{e}' \in \mathrm{E}_{\langle \mathrm{w,t}\rangle} \forall \mathrm{e} \in \mathrm{E}_{\langle \mathrm{w,t}\rangle} \forall \mathrm{x}_1, \ldots, \mathrm{x}_n \psi(\mathrm{e}, \mathrm{x}_1, \ldots, \mathrm{x}_n) \rightarrow \mathrm{e} \subset \mathrm{e}')$

$\mathrm{E}_{\langle \mathrm{w,t}\rangle}$ is the "domain of eventualities in w at t". $\mathrm{Dom}(\langle \mathrm{w,t}\rangle) = \mathrm{D}_{\langle \mathrm{w,t}\rangle} \cup \mathrm{E}_{\langle \mathrm{w,t}\rangle}$.

(iv) $[\![\ ]\!]$ is an interpretation function that assigns to DRS predicates functions δ such that for $\langle \mathrm{w,t}\rangle \in \mathrm{W}\times\mathrm{T}\delta(\langle \mathrm{w,t}\rangle) \in \mathcal{P}(\cup_{n\in\omega}(\mathrm{Dom}(\langle \mathrm{w,t}\rangle))^n)$.
To quantifier relation symbols, $[\![\ ]\!]$ assigned functions γ such that for $\langle \mathrm{w,t}\rangle \in \mathrm{W} \times \mathrm{T}$ into $\gamma(\langle \mathrm{w,t}\rangle) \in \mathcal{P}\mathcal{P}(\cup_{n\in\omega}(\mathrm{Dom}(\langle \mathrm{w,t}\rangle)^n))$, where $\mathrm{Dom}(\langle \mathrm{w,t}\rangle) = \mathrm{D}_{\langle \mathrm{w,t}\rangle} \cup \mathrm{E}_{\langle \mathrm{w,t}\rangle}$.

$\mathrm{e}_1 \oplus \mathrm{e}_2 = \min_{\subset}\{\mathrm{e}' : \mathrm{e}_1 \subset \mathrm{e}'\&\mathrm{e}_2 \subset \mathrm{e}'\}$ if there is such an e′; = 0 otherwise

Define a *proper embedding* of a DRS K *in* a model M *with respect to* a function from discourse referents into objects in $\mathrm{D}_{\mathrm{M}\langle \mathrm{w,t}\rangle}$. Call any such function from discourse referents into D_M an *embedding function*.

Define an embedding function g to *extend* an embedding function f *relative to* K in M at $\langle \mathrm{w,t}\rangle$ (written $g \supseteq_\mathrm{K}$ f) just in case $\mathrm{Dom(g)} = \mathrm{Dom(f)}\cup\mathrm{U_K}, \mathrm{g} \supseteq \mathrm{f}$.

Define an *external anchor for* K *in* M to be a partial function from $\mathrm{U_K}$ into $(\cup_{\langle \mathrm{w,t}\rangle\in\mathrm{W}\times\mathrm{T}}(\mathrm{Dom}(\langle \mathrm{w,t}\rangle)) \cup \mathrm{T}$, such that if $\mathrm{n} \in \mathrm{U_K}$ then A(n) = the utterance time of the discourse, and if $\mathrm{i} \in \mathrm{U_K}\mathrm{A(i)}$ = the speaker of the discourse.

Define a *proper embedding* f *of* K *in* M *at* $\langle \mathrm{w,t}\rangle$ (written $[\mathrm{f,K}]^\mathrm{M}_{\mathrm{w,t}}$) with respect to a possibly empty external anchor A for K in M and satisfaction for a condition in M relative to an embedding function f for the DRS in which the conditions occur at $\langle \mathrm{w,t}\rangle$ (written (M, w, t)$\models_\mathrm{f}$).

SATISFACTION

(i) If ψ is an atomic condition of the form $\phi(\mathbf{x}_1, \ldots, \mathbf{x}_n)$, then (M, w, t)$\models_\mathrm{f} \psi$ iff $\langle \mathrm{f}(\mathbf{x}_1), \ldots, \mathrm{f}(\mathbf{x})_n\rangle \in [\![\phi]\!]^\mathrm{M}_{\mathrm{w,t}}$.

(ii) If ψ is an atomic condition of the form $\mathbf{x}_1 = \mathbf{x}_2$, then M(M, w, t)$\models_\mathrm{f} \psi$ iff $\mathrm{f}(\mathbf{x}_1) = \mathrm{f}(\mathbf{x}_2)$.

(ii) If ψ is a condition of the form $\mathbf{X} = \mathbf{x}_1 + \ldots + \mathbf{x}_n$, then (M, w, t)$\models_\mathrm{f} \psi$ iff $\mathrm{f}(\mathbf{X}) = \mathrm{f}(\mathbf{x}_1) \oplus \ldots \oplus \mathrm{f}(\mathbf{x}_n)$.

(iii) If ψ is a condition of the form $\mathbf{X} = \Sigma\mathbf{x}\mathrm{K}_1$, then (M, w, t)$\models_\mathrm{f} \psi$ iff $\mathrm{f}(\mathbf{X}) = \oplus\{\mathrm{b} \in \mathrm{D}\langle \mathrm{w,t}\rangle : \exists \mathrm{g} \supseteq_{\mathrm{K}_1} \mathrm{f}(\mathrm{g}(\mathbf{x}) = \mathrm{b}\ \&\ [\mathrm{g,K}_1]^\mathrm{M}_{\mathrm{w,t,f}})\}$.

(iv) If ψ is of the form $\mathbf{e}$-K_1, then (M, w, t)$\models_\mathrm{f} \psi$ iff $\exists \mathrm{g} \supseteq_{\mathrm{K}_1} \mathrm{f}[\mathrm{g,K}_1]^\mathrm{M}_{\mathrm{w,t,f}})$.

(v) If ψ is a condition of the form $\mathrm{K}_1 \Rightarrow \mathrm{K}_2$, then (M, w, t)$\models_\mathrm{f} \psi$ iff $\forall \mathrm{g} \supseteq_{\mathrm{K}_1} \mathrm{f}([\mathrm{g,K}_1]^\mathrm{M}_{\mathrm{w,t,f}} \rightarrow \exists \mathrm{h} \supseteq_{\mathrm{K}_2} \mathrm{g}[\mathrm{h,K}_2]^\mathrm{M}_{\mathrm{w,t,g}})$.

(vi) If ψ is a condition of the form $\mathrm{K}_1 \underset{\mathrm{x}}{\overset{\psi}{\Diamond}} \mathrm{K}_2$, then (M, w, t)$\models_\mathrm{f} \psi$ iff $\langle\{\mathrm{b} : \exists \mathrm{g} \supseteq_{\mathrm{K}_1} \mathrm{f}[\mathrm{g,K}_1]^\mathrm{M}_{\mathrm{w,t,f}}\ \&\ \mathrm{g}(\mathbf{x}) = \mathrm{b}\}, \{\mathrm{b} : \exists \mathrm{g} \supseteq_{\mathrm{K}_1} \mathrm{f}[\mathrm{g,K}_1]^\mathrm{M}_{\mathrm{w,t,f}}\ \&\ \exists \mathrm{h} \supseteq_{\mathrm{K}_2} \mathrm{g}[\mathrm{h,K}_2]^\mathrm{M}_{\mathrm{w,t,g}}\mathrm{h}(\mathbf{x}) = \mathrm{b}\}\rangle \in [\![\phi]\!]^\mathrm{M}_{\mathrm{w,t}}$.

(vii) If ψ is a condition of the form $\neg\mathrm{K}_1$, then (M, w, t)$\models_\mathrm{f} \psi$ iff $\neg\exists \mathrm{g} \supseteq_{\mathrm{K}_1} \mathrm{f}[\mathrm{g,K}_1]^\mathrm{M}_{\mathrm{w,t,f}}$.

(viii) If A is an external anchor for K in M, then $[\mathrm{f,K}]^\mathrm{M}_{\mathrm{w,t,g}}$ iff
(i) $\mathrm{f} \supseteq_\mathrm{K}$ g; (ii) $\mathrm{A} \subseteq \mathrm{f}$; (iii) $\forall\theta \in \mathrm{Con_K}(M, w, t)\models_\mathrm{f} \theta$.

(ix) $[\mathrm{f,K}]^\mathrm{M}_{\mathrm{w,t}}$ iff $[\mathrm{f,K}]^\mathrm{M}_{\mathrm{w,t},\emptyset}$.

8.3 Predicate Logic with Anaphora (Dekker 1994, 2012)

- Like DPL, Predicate Logic with Anaphora (PLA; Dekker 1994, 2012) is an extension of PL with compositionality as a primary goal
- Unlike DPL, PLA is an **update semantics**, so it involves the tracking (and growth) of information, as formulas update **information states**
- A PLA information state includes information about subjects:
  "These subjects, composed of the possible values of candidate antecedent terms, are the potential 'referents' of subsequent anaphoric pronouns' (Dekker 1994: 80)
  - These discourse referents are stored in a **list** (an ordered set)
- Rather than treating anaphors as (syntactically free, semantically bound) variables, like DPL, PLA treats anaphors as syntactic objects which index this list of possible subjects in the current info state
- Like in DPL, it is existential quantifiers which introduce new discourse referents:
  A sentence which introduces a(n individual) discourse referent, as in (45a), is translated as containing an existential quantifier, as in (45b)

(45) a. There is a man.
b. $s\,[\![\exists x Mx]\!]_g = \{e \cdot d \mid e \in s\,[\![Mx]\!]_g\,[x/d]\}$
$= \{e \cdot d \mid e \in s \;\&\; d \text{ is a man}\}\ (= s')$ (Dekker 1994: (5))

- It is the existential quantifier which adds a new discourse referent to the information state (extending the list of subjects being tracked, $e \cdot d$)
- This new expanded information state is then constrained, to ensure that the output information state (s') only includes cases in which the most recent subject is a man
- With an info state tracking subjects, PLA has pronouns refer to those subjects by the position in the info state, tracking the order in which they were introduced
- So a pronoun translated as p_0 picks out the most recently introduced subject, p_1 picks out the second most recent subject, etc.
- How we translate an object language pronoun into PLA (p_0 vs. p_1 vs. p_2, etc.) will determine which referent is picked out
- If we interpret (46a) as occurring immediately after (45a), and translate *he* as p_0, then we get the translation and interpretation in (46b)

(46) a. (There is a man.) He walks.
b. $s'\,[\![W\mathrm{p}_0]\!]_g = \{e' \in s' \mid \text{the last element of } e' \text{ walks}\}$
$= \{e \cdot d \mid e \in s \;\&\; d \text{ is a man} \;\&\; d \text{ walks}\}\ (= s'')$ (Dekker 1994: (6))

  - This sentence introduces no new discourse referents—it contains no existential quantifier
  - But it refers back to the list of existing discourse referents, and constrains the information state by only retaining cases where the most recently introduced subject walks
  - Though the argument of W in (46b) picks out the same referent as the argument of (45b)'s M, they are not (bound) variables, as in DPL
  - The pronoun p_0 happens to pick out the same referent, but isn't itself a variable, and isn't bound

Figure 3: Fragment of Predicate Logic with Anaphora (Dekker 1994)

Vocabulary

- set of information states S^n about n subjects $S^n = \mathcal{P}(D^n)$, where $S = \bigcup_{n \in \mathcal{N}} S^n$
- relation constants R^n of arity n
- individual anaphors $A = \{\mathrm{p}_i \mid i \in \mathcal{N}\}$
- individual constants C
- set of terms $T = \{C, V, A\}$
- conjunction $\wedge$
- individual variables V
- negation $\neg$
- existential $\exists$

For a state $s \in S^n$ and $0 < j \leq n$, and for any case $e = \langle d_1, \ldots, d_n \rangle \in s$, d_j is a possible value of the j-th subject of s, and this value will also be indicated as e_j.
If $e \in D^n$ and $e' \in D^m$, then $e \cdot e' = \langle e_1, \ldots, e_n, e'_1, \ldots, e'_m \rangle \in D^{n+m}$
Extension: $e \leq e'$ iff $\exists e'' : e' = e \cdot e''$
For $s \in S^n$ $(i \in D^n)$, N_s $(= N_i)$ $= n$, the number of subjects of s (i)
Update: $s \leq s'$ iff $N_s \leq N_{s'}$ and $\forall e' \in s' \exists e \in s : e \leq e'$

Syntax

The set L of PLA formulas is the smallest set such that:

1. if $t_1, \ldots, t_n \in T$ and $R \in R^n$, then $Rt_1 \ldots t_n \in L$
2. if $t_1, t_2 \in T$, then $t_1 = t_2 \in L$
3. if $\phi \in L$, then $\neg\phi \in L$
4. if $\phi \in L$ and $x \in V$, then $\exists x \phi \in L$
5. if $\phi, \psi \in L$, then $(\phi \wedge \psi) \in L$
   $\forall x \phi$ abbreviates $\neg\exists x \neg\phi$ $\qquad$ $\phi \to \psi$ abbreviates $\neg(\phi \wedge \neg\psi)$

$s\,[\![\phi]\!]_{\mathcal{M},g}$ is the interpretation of ϕ in info state s with respect to model $\mathcal{M}$ and assignment g.
$[\![\phi]\!]_{\mathcal{M},g}$ is a (partial) update function on info states.

Semantics

A PLA model $M = \langle D, F \rangle$ consists of a non-empty domain D of individuals, and an interpretation function F which assigns individuals in D to individual constants and sets of n-tuples of individuals to n-place relation expressions. Constants and variables are evaluated in the usual way with respect to a model and an assignment function. Pronouns are evaluated relative to an information state s and a case $e \in s$.

- $[c]_{\mathcal{M},s,e,g} = F(c)$ for all constants c
- $[x]_{\mathcal{M},s,e,g} = g(x)$ for all variables x
- $[\mathrm{p}_i]_{\mathcal{M},s,e,g} = e_{N_{s-i}}$ for all pronouns p_i and e and s such that $e \in s$ and $N_s > i$

If X is a set of terms, then I_x is the smallest number $\geq$ the index of every pronoun in X.

1. $s\,[\![Rt_1 \ldots t_n]\!]_{\mathcal{M},g} = \{e \in s \mid \langle [t_1]_{\mathcal{M},s,e,g}, \ldots, [t_n]_{\mathcal{M},s,e,g} \rangle \in F(R)\}$ (if $N_s > I_{\{t_1,\ldots,t_n\}}$)
2. $s\,[\![t_1 = t_2]\!]_{\mathcal{M},g} = \{e \in s \mid [t_1]_{\mathcal{M},s,e,g} = [t_2]_{\mathcal{M},s,e,g}\}$ (if $N_s > I_{\{t_1,t_2\}}$)
3. $s\,[\![\neg\phi]\!]_{\mathcal{M},g} = \{e \in s \mid \neg\exists e' : e \leq e' \;\&\; e' \in s\,[\![\phi]\!]_{\mathcal{M},g}\}$
4. $s\,[\![\exists x \phi]\!]_{\mathcal{M},g} = \{e' \cdot d \mid d \in D \;\&\; e' \in s\,[\![\phi]\!]_{\mathcal{M},g[x/d]}\}$
5. $s\,[\![\phi \wedge \psi]\!]_{\mathcal{M},g} = s\,[\![\phi]\!]_{\mathcal{M},g}\,[\![\psi]\!]_{\mathcal{M},g}$

Support: $s \models_{\mathcal{M},g} \phi$ iff $\forall e \in s : \exists e' : e \leq e' \;\&\; e' \in s\,[\![\phi]\!]_{\mathcal{M},g}$
Entailment: $\phi_1, \ldots, \phi_n \models \psi$ iff $\forall \mathcal{M}, g \forall s \in S : s\,[\![\phi_1]\!]_{\mathcal{M},g} \ldots s\,[\![\phi_n]\!]_{\mathcal{M},g} \models_{\mathcal{M},g} \psi$ (if defined)

8.4 Update with Centering (Bittner 2009, 2011)

- Update with Centering (UC; Bittner 2009, 2011), like PLA, is an update semantics, but one which "represents not only changing information but also changing focus of attention in discourse"
- PLA represents attention indirectly, in that some discourse referents are more prominent than others—in particular, the most recent ones
  - But that some object language pronoun like *she* is more likely to be translated as p_0 than as p_7 is a fact about the language, and not a feature of PLA
- UC, in contrast, hard-codes those most prominent positions into the system
- It also vastly expands the number of types which it deals with (not just individuals!)

- In UC, a discourse tracks not only information but also attention, so the knowledge and context against which a sentence is interpreted is a **state of infotention**

  "A state of infotention is a set of *lists* of prominence-ranked semantic objects that can currently antecede discourse anaphors. Refining PLA, a UC-list is structured into a top sub-list of prominence-ranked topical objects (in the current center of attention) and a bottom sub-list of prominence-ranked background objects (currently in the periphery)" (Bittner 2009: 2)
- Just as in PLA, we extend our state by adding new discourse referents and constrain it by ensuring that certain properties hold of referents under discussion
- And our attention can be modified just as our information can, so Bittner 2009 also introduces operators which reorder these lists
- Where PLA had a single list, UC has pairs of lists, where each pair describes a way (the assignments on) the world could be
- Those lists now contain more than just individuals: they also contain events, states, times, worlds, and propositions
- UC does not abstract over tense the way many systems (including DPL and PLA) do, and it also is sensitive to the eventive/stative status of verbs
- Along with the two lists being managed, this makes the UC translation of a sentence much more comprehensive than its DPL or PLA equivalent would be
- For example, Bittner 2009: (4) gives the sentence in (47a) the UC translation in (47b), which reduces to (47c)

(47) a. Jim is busy.

b. $^{\top}[x|x =_i jim]\ ^{\top};\ (^{\mathrm{P}}[\vartheta_{\top\omega}\top\varepsilon \leq_i \top\tau]; [\top\omega \in \top\omega\|]; ([s|busy_{\top\omega}\langle s, \textsc{ctr}\ s\rangle]\ ^{\bot};$
$[\textsc{at}_{\top\omega}\langle\bot\sigma, \top\tau\rangle,\ \textsc{ctr}\ \bot\sigma =_i\ \top\delta]));\ ^{\top}[p|p = \top\omega\|]$

c. $^{\top}[x|x =_i jim]\ ^{\top};\ ^{\mathrm{P}}[\vartheta_{\top\omega}\top\varepsilon \leq_i \top\tau]; [s|busy_{\top\omega}\langle s, \top\delta\rangle, \top\tau \subset_i \vartheta_{\top\omega}s];\ ^{\top}[p|p = \top\omega\|]$

- This may look arcane, but we can go through it step by step

1. The first box, $^{\top}[x|x =_i jim]$ introduces Jim into the discourse in the $\top$-list, making him a topical individual (and in fact the most topical individual, being at the top of the $\top$-list)
2. The second box, $^{\mathrm{P}}[\vartheta_{\top\omega}\top\varepsilon \leq_i \top\tau]$, represents the non-past tense presupposition carried by *is*, and ensures that the topic time ($\top\tau$) is no earlier than the perspective point (here, by default, the speech act time $\top\varepsilon$)
   - No topic time has yet been introduced, so it is still the speech time, so this is satisfied

3. The third box, $[\top\omega \in \top\omega\|]$, represents the modality of the predicate, and asserts that the world of evaluation (the most prominent world $\top\omega$) is in the common ground ($\top\omega\|$)

    - This condition "is trivially true in root clauses, where the evaluation world is the topic world" (Bittner 2009: 8).

4. The fourth box, $[s|busy_{\top\omega}\langle s, \textsc{ctr}\ s\rangle]$, adds a state of being *busy* to the background list

5. The arguments of this state (its world, time and individual) are added by the fifth box, which identifies its world and time with the topic world and time ($\textsc{at}_{\top\omega}\langle\perp\sigma, \top\tau\rangle$) and its center with the topical individual ($\textsc{ctr}\ \perp\sigma =_i\ \top\delta$)

6. The sixth and final box takes all of the topic worlds which meet this description and identifies them with a topical proposition ($[p|p = \top\omega\|]$)

    - This is then a packaged representation of all of the updates herein, which we can use to refer back to this whole complex

- As we can see, unlike DPL and PLA, UC is compositional not just at the sentence level, but at the morpheme level
    - The tense presupposition and modal assertion of *is* are represented as distinct updates, which in English are both part of the lexical meaning of *is*
    - But we might expect these to be associated with specific morphemes in a language with richer verbal morphology; see Bittner 2009 for UC handling morphological encoding of mood in Kalaallisut
    - The simplified translation in (47c) effectively has one update associated with each word, plus an update for the proposition associated with the sentence which Bittner 2009 associates with the "full stop prosody" of the declarative mood
- The example in (47) also demonstrates how much of the UC system makes use of anaphoric relations
    - Every occurrence of $\top$ or $\perp$ references the topic lists, either updating them or checking new information against them
    - Not just for individuals, but also for tense and modality, formalizing the observations of Partee 1973; Stone 1997
    - Even the naming of new propositional variables is anaphoric, as we saw with the final update of (47b/c), which refers back to the current set of topic worlds, and assigns that set a propositional variable
- As one might expect, Bittner 2009 translates English pronouns as anaphors
    - *I* and *you* are the center and experiencer, respectively, of the topical speech act event
    - A third person pronoun like *she* is just the topical individual, either foregrounded ($\top\delta$) or backgrounded ($\perp\delta$)
        * So like PLA, resolving such an anaphor will be sensitive to recency (within each list)
        * But it isn't subject to the arbitrary translation of a pronoun into p_0 or p_4, etc.

Figure 4: Fragment of Update with Centering (Bittner 2009)

Vocabulary

- $\langle D\rangle^{n,m} = D^n \times D^m$ is the set of $\top\bot$-lists of n topical objects and m background objects
- For any $\top\bot$-list $\mathsf{i} = \langle \mathsf{i}_1, \mathsf{i}_2\rangle \in \langle D\rangle^{n,m}$, $\top\mathsf{i} = \mathsf{i}_1$ and $\bot\mathsf{i} = \mathsf{i}_2$. Thus, $\mathsf{i} = \langle \top\mathsf{i}, \bot\mathsf{i}\rangle$.
- An n, m-*infotention state* is any subset of $\langle D\rangle^{n,m}$. The null set, $\emptyset$, is the *absurd state*.

Discourse deferents for propositions (type ωt), worlds (ω), individuals, (δ), events (ε), states (σ), times (τ). These compose $(\Theta|5)$. A $\top\bot$-list is an object of type s.

The set of UC *types* Θ is the smallest set such that

(i) $\{t, \omega, \delta, \varepsilon, \sigma, \tau\} \subseteq \Theta$, (ii) $(ab) \in \Theta$, and (iii) $s \in \Theta$.

- common ground p_0
- run time ϑ
- temporal precedence $<$
- beginning BEG
- end END
- speech event e_0
- place π
- consequent state CON
- experiencer DAT
- center CTR

Syntax

Define for each type $a \in \Theta$ the set of a-terms as follows:

1. $Con_a \cup Var_a \subseteq Term_a$
2. $\lambda u_a(B) \in Term_{ab}$, if $u_a \in Var_a$ and $B \in Term_b$
3. $BA \in Term_b$, if $B \in Term_{ab}$ and $A \in Term_a$
4. $\neg A, (A \rightarrow B), (A \wedge B), (A \vee B) \in Term_t$, if $A, B \in Term_t$
5. $\forall u_a B, \exists u_a B \in Term_t$, if $u_a \in Var_a$ and $B \in Term_t$
6. $(A = B) \in Term_t$, if $A, B \in Term_a$
7. $(u_a{}^{\top}\!\oplus B), (u_a{}^{\bot}\!\oplus B) \in Term_s$, if $a \in (\Theta|5)$, $u_a \in Var_a$, and $B \in Term_s$
8. $\top a, \bot a \in Term_{sa}$, if $a \in (\Theta|5)$
9. $A\{B\} \in Term_{at}$, if $a \in (\Theta|5)$, $A \in Term_{sa}$, and $B \in Term_{st}$
10. $(A\ ;\ B), (A\ {}^{\top}\!;\ B), (A\ {}^{\bot}\!;\ B) \in Term_{(st)st}$, if $A, B \in Term_{(st)st}$
11. $(A \subset B), (A < B) \in Term_t$, if $A, B \in Term_t$
12. CON $A \in Term_\sigma$, if $A \in Term_\varepsilon$
    BEG A, END $A \in Term_\varepsilon$, if $A \in Term_\sigma$
    CTR A, DAT $A \in Term_\delta$, if $A \in Term_\varepsilon \cup Term_\sigma$
13. $\vartheta(W, A) \in Term_\tau$, if $W \in Term_\omega$ and $A \in Term_\varepsilon \cup Term_\sigma$
    $\pi(W, A) \in Term_\delta$, if $W \in Term_\omega$ and $A \in Term_\varepsilon \cup Term_\sigma$

Figure 4, continued

FRAMES

A UC frame is a set $\{D_a | a \in \Theta\}$ of non-empty pairwise disjoint sets where:

(i) $D_t = \{1, 0\}$, D_τ is the set of non-empty convex sets of integers;

(ii) $D_a, b = \{f | \emptyset \subset \mathrm{Dom} f \subseteq D_a \wedge \mathrm{Ran} f \subseteq D_b\}$; and

(iii) $D_s \cup_{n,m \geq 0} \langle D \rangle^{n,m}$, with $D = \cup_{a \in (\Theta | 5)} D_a$.

$^{\{\}}f$ is the set characterized by function f $\quad$ $^{\chi}A$ is the characteristic function of set A

MODELS

A UC-model is a structure $M = \langle \{D_a | a \in \Theta\}, <_t, \mathsf{p}_0, \mathsf{e}_0, [\![\cdot]\!] \rangle$, where:

(i) $\{D_a | a \in \Theta\}$ is a UC frame;

(ii) for all $\mathsf{t}, \mathsf{t}' \in D_\tau$, $\mathsf{t} <_\tau \mathsf{t}'$ iff $\forall n \in \mathsf{t} \forall n' \in \mathsf{t}' : n < n'$;

(iii) $\mathsf{p}_0 \in D_{\omega t} \backslash \{\emptyset\}$ and $\mathsf{e}_0 \in D_\varepsilon$; and

(iv) $[\![\cdot]\!]$ assigns to each $A \in Con_a$ a value $[\![A]\!] \in D_a$ and to each $B \in \{\text{CON}, \text{BEG}, \text{END}, \text{CTR}, \text{DAT}, \vartheta, \pi\}$ a value $[\![B]\!]$ such that:

(a) $[\![\text{CON}]\!] \in D_{\varepsilon\sigma}, [\![\text{BEG}]\!], [\![\text{END}]\!] \in D_{\sigma\varepsilon}, [\![\text{CTR}]\!], [\![\text{DAT}]\!] \in \{f_\varepsilon \cup f_\sigma | f_\varepsilon \in D_{\varepsilon\delta} \wedge f_\sigma \in D_{\sigma\delta}\}$,
$[\![\vartheta]\!] \in \{f_\varepsilon \cup f_\sigma | f_\varepsilon \in D_{\omega\varepsilon\tau} \wedge f_\sigma \in D_{\omega\sigma\tau}\}, [\![\pi]\!] \in \{f_\varepsilon \cup f_\sigma | f_\varepsilon \in D_{\omega\varepsilon\delta} \wedge f_\sigma \in D_{\omega\sigma\delta}\}$

(b) $\forall \mathsf{w} \in D_\omega, \mathsf{a} \in D_\delta, \mathsf{e} \in D_\varepsilon, \mathsf{s} \in D_\sigma, \mathsf{ev} \in D_\varepsilon \cup D_\sigma, \mathsf{t} \in D_\tau$:
$[\![\vartheta]\!](\mathsf{w},\mathsf{e}) = \mathsf{t} \rightarrow \exists n : \mathsf{t} = \{n\} \wedge [\![\vartheta]\!](\mathsf{w}, [\![\text{BEG}]\!]([\![\text{CON}]\!](\mathsf{e}))) = \{(n+1)\}$
$[\![\vartheta]\!](\mathsf{w},\mathsf{s}) = \mathsf{t} \rightarrow \{\min_< \mathsf{t}\} = [\![\vartheta]\!](\mathsf{w}, [\![\text{BEG}]\!](\mathsf{s})) <_\tau [\![\vartheta]\!](\mathsf{w}, [\![\text{END}]\!](\mathsf{s})) = \{\max_< \mathsf{t}\}$

(c) $[\![\text{CTR}]\!](\mathsf{ev}) = \mathsf{a} \rightarrow [\![\text{CTR}]\!]([\![B]\!](\mathsf{ev})) = \mathsf{a}$ $\quad$ for $B \in \{\text{CON, BEG, END}\}$
$\langle \mathsf{ev}, \mathsf{a}, \ldots \rangle \in {}^{\{\}}[\![A]\!](\mathsf{w}) \rightarrow [\![\text{CTR}]\!](\mathsf{ev}) = \mathsf{a}$ $\quad$ for $A \in Con_{\omega\varepsilon\delta\ldots t} \cup Con_{\omega\sigma\delta\ldots t}$

(d) $\exists \mathsf{t} \forall \mathsf{w} \in {}^{\{\}}\mathsf{p}_0 : t = [\![\vartheta]\!](\mathsf{w}, \mathsf{e}_0) \wedge \langle \mathsf{e}_0, [\![\text{CTR}]\!](\mathsf{e}_0) \rangle \in {}^{\{\}}[\![spk]\!](\mathsf{w})$

The pair $\langle \mathsf{p}_0, \mathsf{e}_0 \rangle$ is the *utterance context* of M.

The *default infotention state* ${}^*\langle \mathsf{p}_0, \mathsf{e}_0 \rangle := {}^{\chi}\{\langle \langle \mathsf{t}, \mathsf{w}, \mathsf{p}_0, \mathsf{e}_0 \rangle, \langle\ \rangle \rangle \mid \mathsf{w} \in {}^{\{\}}\mathsf{p}_0 \wedge \mathsf{t} = [\![\vartheta]\!](\mathsf{w}, \mathsf{e}_0)\}$

ABBREVIATIONS

i.	$(\mathsf{x})_n$	$=$	the nth coordinate, x_n	for $\mathsf{x} \in D^{n+m}, n \geq 1$
	$(\mathsf{x})_a$	$=$	the subsequence consisting of $\mathsf{x}_i \in D_a$	for $\mathsf{x} \in D^n, a \in \Theta$
ii.	$(\mathsf{d} \oplus \mathsf{x})$	$=$	$\langle \mathsf{d}, \mathsf{x}_1, \ldots, \mathsf{x}_n \rangle$	for $\mathsf{d} \in D, \mathsf{x} \in D^n$
	$\mathsf{y} > \mathsf{x}$	iff	$\mathsf{y} = (\mathsf{y}_1 \oplus \ldots (\mathsf{y}_n \oplus \mathsf{x}))$	for $\mathsf{x} \in D^n, \mathsf{y} \in D^{n+m}$
	$\mathsf{y} \geq \mathsf{x}$	iff	$\mathsf{y} > \mathsf{x} \vee \mathsf{y} = \mathsf{x}$	for $\mathsf{x} \in D^n, \mathsf{y} \in D^m$

Figure 4, continued

Semantics

$X \doteq Y$ reads as 'X is Y if Y is defined, else X is undefined'

i.	$\llbracket A \rrbracket^g$	$=$	$\llbracket A \rrbracket$	for any $A \in Con_a$
	$\llbracket u \rrbracket^g$	$=$	$g(u)$	for any $u \in Var_a$
ii.	$\llbracket \lambda u_a(B) \rrbracket^g(\mathsf{d})$	$\doteq$	$\llbracket B \rrbracket^{g[u/\mathsf{d}]}$	for any $\mathsf{d} \in D_a$
iii.	$\llbracket BA \rrbracket^g$	$\doteq$	$\llbracket B \rrbracket^g(\llbracket A \rrbracket^g)$	
iv.	$\llbracket \neg A \rrbracket^g$	$\doteq$	$1 \backslash \llbracket A \rrbracket^g$	
	$\llbracket (A \rightarrow B) \rrbracket^g$	$\doteq$	$1 \backslash (\llbracket A \rrbracket^g \backslash \llbracket B \rrbracket^g)$	
	$\llbracket (A \wedge B) \rrbracket^g$	$\doteq$	$\llbracket A \rrbracket^g \cap \llbracket B \rrbracket^g$	
	$\llbracket (A \vee B) \rrbracket^g$	$\doteq$	$\llbracket A \rrbracket^g \cup \llbracket A \rrbracket^g$	
v.	$\llbracket \forall u_a A \rrbracket^g$	$\doteq$	$\bigcap_{\mathsf{d} \in D_a} \llbracket A \rrbracket^{g[u/\mathsf{d}]}$	
	$\llbracket \exists u_a A \rrbracket^g$	$\doteq$	$\bigcup_{\mathsf{d} \in D_a} \llbracket A \rrbracket^{g[u/\mathsf{d}]}$	
vi.	$\llbracket A_a = B_a \rrbracket^g$	$\doteq$	$\lvert\{\langle \mathsf{d}, \mathsf{d}' \rangle \in {D_a}^2 \mid \mathsf{d} = \llbracket A \rrbracket^g \wedge \mathsf{d}' = \llbracket B \rrbracket^g \wedge \mathsf{d} = \mathsf{d}'\}\rvert$	
vii.	$\llbracket {u_a}^{\top}\!\oplus B \rrbracket^g$	$\doteq$	$\langle (g(u_a) \oplus \top\llbracket B \rrbracket^g), \bot\llbracket B \rrbracket^g \rangle$	
	$\llbracket {u_a}^{\bot}\!\oplus B \rrbracket^g$	$\doteq$	$\langle \top\llbracket B \rrbracket^g, (g(u_a) \oplus \bot\llbracket B \rrbracket^g) \rangle$	
viii.	$\llbracket \top a \rrbracket^g(\mathsf{i})$	$\doteq$	$((\top\mathsf{i})_a)_1$	for any $\mathsf{i} \in D_s$
	$\llbracket \bot a \rrbracket^g(\mathsf{i})$	$\doteq$	$((\bot\mathsf{i})_a)_1$	for any $\mathsf{i} \in D_s$
ix.	$\llbracket A\{B\} \rrbracket^g$	$\doteq$	${}^{\chi}\{\llbracket A \rrbracket^g(\mathsf{i}) \mid \mathsf{i} \in^{\{\}} \llbracket B \rrbracket^g\}$	
x.	$\mathsf{c}\llbracket A; B \rrbracket^g$	$\doteq$	$\mathsf{c}\llbracket A \rrbracket^g \llbracket B \rrbracket^g$	for any $\mathsf{c} \in D_{st}$
	$\mathsf{c}\llbracket A\ {}^{\top}\!;\ B \rrbracket^g$	$\doteq$	$\{\mathsf{l} \in \mathsf{c}\llbracket A; B \rrbracket^g \mid \exists a \forall \mathsf{k} \in \mathsf{c}\llbracket A; B \rrbracket^g\ \exists \mathsf{j} \in \mathsf{c}\llbracket A; B \rrbracket^g\ \exists \mathsf{i} \in c\ \exists \mathsf{d} \in D_a :$ $\top\mathsf{k} \geq \top\mathsf{j} > \top\mathsf{i} \wedge (\top\mathsf{j})_1 = \mathsf{d} \wedge \llbracket B \rrbracket^g \neq \llbracket B[\top a/\bot a] \rrbracket^g \wedge \llbracket \top a \rrbracket^g(\mathsf{k}) = d\}$	
	$\mathsf{c}\llbracket A\ {}^{\bot}\!;\ B \rrbracket^g$	$\doteq$	$\{\mathsf{l} \in \mathsf{c}\llbracket A; B \rrbracket^g \mid \exists a \forall \mathsf{k} \in \mathsf{c}\llbracket A; B \rrbracket^g\ \exists \mathsf{j} \in \mathsf{c}\llbracket A; B \rrbracket^g\ \exists \mathsf{i} \in c\ \exists \mathsf{d} \in D_a :$ $\bot\mathsf{k} \geq \bot\mathsf{j} > \bot\mathsf{i} \wedge (\bot\mathsf{j})_1 = \mathsf{d} \wedge \llbracket B \rrbracket^g \neq \llbracket B[\bot a/\top a] \rrbracket^g \wedge \llbracket \bot a \rrbracket^g(\mathsf{k}) = d\}$	
xi.	$\llbracket A \subset B \rrbracket^g$	$\doteq$	$\lvert\{\langle \mathsf{t}, \mathsf{t}' \rangle \in {D_\tau}^2 \mid \mathsf{t} = \llbracket A \rrbracket^g \wedge \mathsf{t}' = \llbracket B \rrbracket^g \wedge \mathsf{t} \subset \mathsf{t}'\}\rvert$	
	$\llbracket A < B \rrbracket^g$	$\doteq$	$\lvert\{\langle \mathsf{t}, \mathsf{t}' \rangle \in {D_\tau}^2 \mid \mathsf{t} = \llbracket A \rrbracket^g \wedge \mathsf{t}' = \llbracket B \rrbracket^g \wedge \mathsf{t} <_\tau \mathsf{t}'\}\rvert$	
xii.	$\llbracket BA \rrbracket^g$	$\doteq$	$\llbracket B \rrbracket^g(\llbracket A \rrbracket^g)$	
xiii.	$\llbracket B(W, A) \rrbracket^g$	$\doteq$	$\llbracket B \rrbracket^g(\llbracket W \rrbracket^g, \llbracket A \rrbracket^g)$	

Given a state c, an $(st)st$ term K adds the *primary topics* $\top_{\mathsf{c}}K = \{(\top\mathsf{j})_1 \mid \forall g : \mathsf{j} \in^{\{\}} (\mathsf{c}\llbracket K \rrbracket^g) \wedge \mathsf{j} \notin^{\{\}} \mathsf{c}\}$.

K is *true* in c at w iff $\exists \mathsf{p} \in D_\Omega : \top_{\mathsf{c}}K = \{\mathsf{p}\} \wedge \mathsf{w} \in^{\{\}} \mathsf{p}$;

K is *false* in c at w iff $\exists \mathsf{p} \in D_\Omega : \top_{\mathsf{c}}K = \{\mathsf{p}\} \wedge \mathsf{w} \notin^{\{\}} \mathsf{p}$; else K does not have a truth value.

Takeaways from our Whirlwind Tour

- There are a number of theories already available to be used 'off the shelf'
- These different theories are motivated by different concerns, make different assumptions, have different strengths and weaknesses
- Once we get a handle on a framework, we can start to answer the questions we were compiling yesterday (the ones motivated by our puzzles)
  - How would we handle bishops in PLA?
  - How does DRT model Partee's bathroom?
- We might eventually as a field coalesce around the One True Framework which handles everything (doesn't under/overgenerate, has total empirical coverage, etc.)
- Until then, we have to be to able to consider and compare multiple frameworks, to find out what different predictions they make (so as to hopefully eventually decide among them)
- Out of the blue, formalism can be intimidating, but if you take things one piece at a time, you can understand anything!

References

Ariel, Mira. 1999. Cognitive universals and linguistic conventions: The case of resumptive pronouns. *Studies in Language* 23(2). 217–269.

Asher, Nicholas. 1993. *Reference to abstract objects in English: a philosophical semantics for natural language metaphysics*, vol. 50 Studies in Linguistics and Philosophy. Kluwer Academic Publishers.

Asudeh, Arshia. 1998. *Anaphora and argument structure: Topics in the syntax and semantics of reflexives and reciprocals*. University of Edinburgh MA thesis.

Bittner, Maria. 2009. Tense, mood, and centering. Ms. Rutgers University. https://philpapers.org/archive/BITTMA.pdf.

Bittner, Maria. 2011. Time and modality without tenses or modals. In Renate Musan & Monika Rathert (eds.), *Tense across Languages*, 147–188. Niemeyer.

Bresnan, Joan. 2001. *Lexical Functional Syntax*. Oxford: Blackwell.

Carlson, Gregory N. 1977. A unified analysis of the English bare plural. *Linguistics and Philosophy* 1(3). 413–457.

Chomsky, Noam. 1981. *Lectures on Government and Binding*. Foris.

Chomsky, Noam. 1982. *Some concepts and consequences of the theory of government and binding*, vol. 6. MIT press.

Chomsky, Noam. 1986. *Barriers*. MIT Press.

Clark, Herbert H. 1975. Bridging. In *Theoretical Issues in Natural Language Processing (TINLAP*, 169–174. Association for Computational Linguistics.

Clark, Herbert H. & Catherine R. Marshall. 1981. Definite reference and mutual knowledge. In Aravind Joshi, Bonnie Webber & Ivan Sag (eds.), *Elements of Discourse Understanding*, 10–63. Cambridge University Press.

Cosse, Michael. 1996. Indefinite associative anaphora in French. In Christina Hellman & Kari Fraurud (eds.), *Proceedings of the IndiAna Workshop on Indirect Anaphora*, University of Lancaster.

Dalrymple, Mary. 1993. *The Syntax of Anaphoric Binding* 36. Center for the Study of Language (CSLI).

Dalrymple, Mary. 2001. *Lexical Functional Grammar*, vol. 34. Brill.

Dekker, Paul. 1994. Predicate logic with anaphora. In Mandy Harvey & Lynn Santelmann (eds.), *Semantics and Linguistic Theory (SALT)*, vol. 4, 79–95. Cornell Linguistics Circle.

Dekker, Paul. 2012. *Dynamic Semantics*, vol. 91 Studies in Linguistics and Philosophy. Springer.

Elbourne, Paul. 2005. *Situations and Individuals*. MIT Press.

Elbourne, Paul. 2010. On bishop sentences. *Natural Language Semantics* 18(1). 65–78.

Evans, Gareth. 1977. Pronouns, quantifiers, and relative clauses (i). *Canadian Journal of Philosophy* 7(3). 467–536.

Geach, Peter Thomas. 1962. Reference and generality. Cornell Press.

Geurts, Bart & Rob Van der Sandt. 2004. Interpreting focus. *Theoretical Linguistics* 30. 1–44.

Ginzburg, Jonathan & Ivan Sag. 2000. *Interrogative Investigations: the Form, Meaning and Use of English Interrogatives*. CSLI Publications.

Groenendijk, Jeroen & Martin Stokhof. 1991. Dynamic predicate logic. *Linguistics and Philosophy* 14(1). 39–100.

Hahn, Udo, Katja Markert & Michael Strube. 1996. A conceptual reasoning approach to textual ellipsis. In *European Conference on Artificial Intelligence* 12, 572–576.

Hankamer, Jorge & Ivan Sag. 1976. Deep and surface anaphora. *Linguistic Inquiry* 7(3). 391–428.

Heim, Irene. 1990. E-type pronouns and donkey anaphora. *Linguistics and Philosophy* 13(2). 137–177.

Heim, Irene & Angelika Kratzer. 1998. *Semantics in Generative Grammar*, vol. 13. Blackwell Oxford.

Hellman, Christina & Kari Fraurud (eds.). 1996. *Proceedings of the IndiAna Workshop on Indirect Anaphora*. University of Lancaster.

Jaeger, T Florian. 2004. Binding in picture NPs revisited: Evidence for a semantic principle of extended argument-hood. In Miriam Butt & Tracy Holloway King (eds.), *Proceedings of the LFG04 Conference*, CSLI Publications.

Kadmon, Nirit. 1987. *On the unique and non-unique reference and asymmetric quantification*: University of Massachusetts, Amherst dissertation.

Kamp, Hans. 1981. A theory of truth and semantic representation. In Jeroen Groenendijk, Theo Janssen & Martin Stokhof (eds.), *Formal Methods in the Study of Language, Part I*, 277–322. CWI.

Karttunen, Lauri. 1969. Discourse referents. In *Proceedings of the 1969 Conference on Computational Linguistics*, 1–38. Association for Computational Linguistics.

Krifka, Manfred. 2013. Response particles as propositional anaphors. In Todd Snider (ed.), *Semantics and Linguistic Theory (SALT)*, vol. 23, 1–18. CLC Publications. doi:10.3765/salt.v20i0.2584.

Kroll, Nicky. 2008. On bishops and donkeys. *Natural Language Semantics* 16. 359–372.

Link, Godehard. 1983. The logical analysis of plurals and mass terms: A lattice-theoretical approach. In Rainer Bäuerle, Christoph Schwarze & Arnim von Stechow (eds.), *Meaning, Use, and Interpretation of Language*, 302–323. Berlin: Walter de Gruyter.

Luperfoy, Susann. 1992. The representation of multimodal user interface dialogues using discourse pegs. In *Annual Meeting of the Association for Computational Linguistics* 30, 22–31. Association for Computational Linguistics University of Delaware.

Not, Elena, Lucia Tovena & Massimo Zancanaro. 1999. Positing and resolving bridging anaphora in deverbal NPs. In *Workshop on the Relationship between Discourse/Dialogue Structure and Reference*, Association for Computational Linguistics.

Partee, Barbara H. 1973. Some structural analogies between tenses and pronouns in English. *The Journal of Philosophy* 601–609.

Partee, Barbara H. 1984. Nominal and temporal anaphora. *Linguistics and Philosophy* 7(3). 243–286.

Partee, Barbara H. 1989. Binding implicit variables in quantified contexts. In Caroline Wiltshire, Bradley Music & Randolph Graczyk (eds.), *Chicago Linguistic Society*, vol. 25 1, 342–365.

Pollard, Carl & Ivan A Sag. 1992. Anaphors in English and the scope of binding theory. *Linguistic Inquiry* 23(2). 261–303.

Pollard, Carl & Ivan A Sag. 1994. *Head-Driven Phrase Structure Grammar*. University of Chicago Press.

Postal, Paul M. 1969. Anaphoric islands. In *Chicago Linguistic Society*, vol. 5, 205–239.

Prince, Ellen F. 1992. The ZPG letter: Subjects, definiteness, and information-status. In Susan Thompson & William Mann (eds.), *Discourse Description: Diverse Analyses of a Fundraising Text*, 295–325. John Benjamins.

Reinhart, Tanya. 1983. Coreference and bound anaphora: A restatement of the anaphora questions. *Linguistics and Philosophy* 47–88.

Roberts, Craige. 1989. Modal subordination and pronominal anaphora in discourse. *Linguistics and Philosophy* 12(6). 683–721.

Roelofsen, Floris & Donka Farkas. 2015. Polarity particle responses as a window onto the interpretation of questions and assertions. *Language* 19(2). 359–414. doi:10.1353/lan.2015.0017.

Ross, John Robert. 1967. *Constraints on variables in syntax*: Massachusetts Institute of Technology dissertation.

Snider, Todd. 2017. *Anaphoric reference to propositions*: Cornell University dissertation.

Stone, Matthew. 1997. The anaphoric parallel between modality and tense. Ms.

Veltman, Frank. 1996. Defaults in update semantics. *Journal of Philosophical Logic* 25(3). 221–261.

Webber, Bonnie Lynn, Matthew Stone, Aravind Joshi & Alistair Knott. 2003. Anaphora and discourse structure. *Computational Linguistics* 29(4). 545–587.